Financial Frictions, Used Capital, and Misallocation

Job Market Paper

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Motivation

The Big Picture

- Poor countries produce far less output than rich countries
  - not only because they have **fewer** machines and less education
  - but also because they use what they have **less efficiently**: **lower productivity (TFP)**
- Why less efficiently? capital does not go to the firms that would use it best
  - productive firms stay small, and some are never started (misallocation)
- **Key question:** why don't productive firms in poor countries get the capital they need to grow?

The answer matters for productivity, entrepreneurship, and development.

Motivation

- Many frictions keep capital away from productive firms
  - regulations, weak institutions, costs of adjusting, **financial frictions**
- I focus on **financial frictions**
  - talented but poor entrepreneurs cannot get enough capital
  - in the literature, capital is **homogeneous**: a constrained firm can only **shrink**
  - but data show that young, small firms also change the **type** of capital: they use more **second-hand machines**
- **This paper:** What is the role of used capital in an economy with financial frictions?
  - **Why** do constrained firms choose used capital?
  - **How** does it affect aggregate productivity and who becomes an entrepreneur?

Motivating Evidence

Small and constrained firms Eisfeldt & Rampini (2007), US Census

- Used capital is **over 25%** of investment for the smallest firms, **about 10%** for the largest

Young firms Ma, Murfin & Pratt (2022), US equipment-collateral records

- Start-ups buy old equipment from mature firms; where more is available, **more start-ups enter**

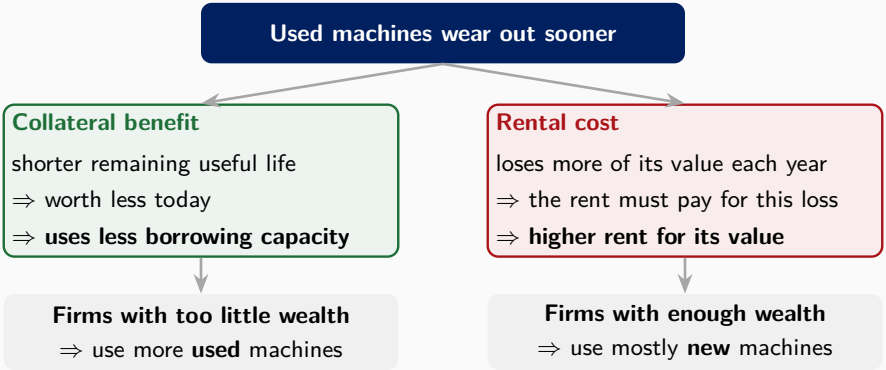
Small and young firms rely on used capital, and it helps them start.

This Paper

1. Document **descriptive evidence**: young firms use more second-hand equipment
2. Provide **micro-level evidence**: when a firm faces a credit constraint, does it use a larger share of **second-hand machines**?
   - follow the same Vietnamese firms over time, 2005–2015
   - measure constraints directly from survey questions on loan problems
3. Develop a **model** of financial frictions with new and used capital
   - firms **rent** a **mix** of new and used machines (imperfect substitutes)
   - borrowing constraint: how much capital a firm can rent depends on its wealth
   - used capital **depreciates faster** than new capital (explained on the next slide)
4. **Quantitatively assess** the role of used capital for aggregate productivity and who becomes an entrepreneur, using Vietnamese firm data

The Mechanism in One Slide

Each firm has a **borrowing capacity**: it can rent machines only up to a value set by its **wealth**



What I Find

- Using micro-data from Vietnam, I document that young firms use a larger share of **second-hand equipment**: 31% vs. 22% for older firms
- When firms face credit constraints, they use more second-hand machines, especially when they lack collateral. As young firms grow, they shift toward new machines.
- In my model, firms with too little wealth tilt toward used capital because it **requires less collateral**; firms with enough wealth use mostly new capital
- Quantitatively, access to used capital:
  - helps talented firms grow, and raises the share of talented but poor people who start a firm from **34% to 42%**
  - so machines and workers are used more efficiently: it removes **39%** of the productivity loss from financial frictions

Contribution to the Literature

Misallocation and financial frictions

Restuccia & Rogerson (2008); Hsieh & Klenow (2009); Buera, Kaboski & Shin (2011); Moll (2014)

Standard models study *how much* capital constrained firms use.

I also study *which type* of capital they use: new or used.

Entrepreneurship and wealth

Evans & Jovanovic (1989); Cagetti & De Nardi (2006); Buera & Shin (2013)

Used capital can change who enters entrepreneurship, not only firm scale.

Used capital and financing

Eisfeldt & Rampini (2007); Rampini (2019); Ma, Murfin & Pratt (2022)

Older assets require less financing because their market value is lower.

I bring this mechanism into a general-equilibrium model and quantify it using Vietnamese panel data.

Evidence on Used Capital

Data

Sample

- Manufacturing firms, surveyed every two years: **5,131 firms** and **15,811 firm-year observations**

Key variables

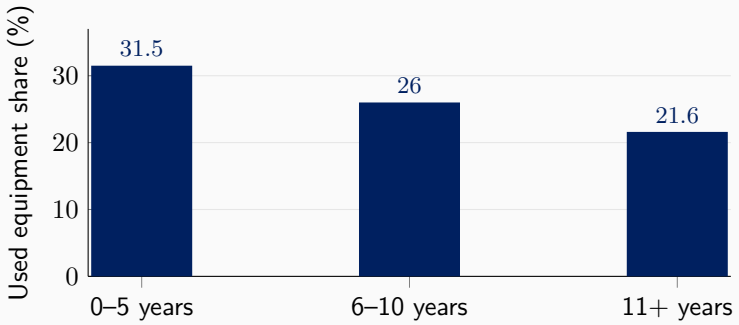
- Whether equipment is bought **new, used, or self-built**; its age; assets; debt; and credit access

Why Vietnam?

- Vietnam: **limited access to financing**
- Used equipment is an important alternative for credit-constrained firms
- An active second-hand market lets firms get equipment without paying for new machines
- About **a quarter** of firms' main equipment is second-hand

The data let me observe both firms' financial position and their choice between new and used machines.

Younger Firms Use More Used Equipment



Firms aged 0–5 use about 10 percentage points more used equipment than firms aged 11+.

Three Facts from the Data

- **Credit constraint**, measured directly from the survey:
  - applied for a loan and had a problem getting it, or was denied
  - did not apply because of too little collateral or a hard process
- **Fact 1:** Who is constrained?
- **Fact 2:** When a firm is constrained, does it use more second-hand machines?
- **Fact 3:** As young firms grow, do they move to new machines?

Each fact matches one prediction of the model.

Fact 1: Young and Asset-Poor Firms Are More Often Constrained

I compare firms of the **same size**, in the **same sector, province and year**

Firm	Chance of being credit constrained
compared with a young firm (0–5 years):	
aged 6–10 years	↓ **lower**
aged 11+ years	↓ **even lower**
compared with a firm of the same age:	
with **more assets**	↓ **lower**

▸ Regression ▸ Results

Fact 2: When Credit Is Tight, Firms Use More Second-Hand Machines

I follow the **same firm over time** and compare its years

(holding the firm's size, assets and age fixed)

Firm's situation	**Share of second-hand machines**
compared with years it is not constrained:	
credit constrained	↑ **higher**
constrained, already has a loan	↑ **higher**
constrained, no loan	— no clear change
compared with years it got its loan without problems:	
loan problem: **lack of collateral**	↑ **much higher**
loan problem: other reasons	— no clear change

Firms shift to second-hand machines when collateral limits their borrowing.

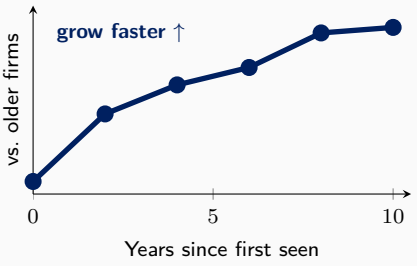
▸ Regression ▸ Results ▸ Timing ▸ Timing results ▸ Robustness ▸ Zero debt ▸ Checks

Fact 3: As Young Firms Grow, They Move to New Machines

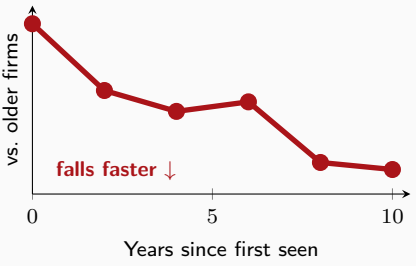
I follow **young firms** (first seen at age 0–5) over 10 years

and compare them with **older firms** in the same years

Assets



Share of second-hand machines



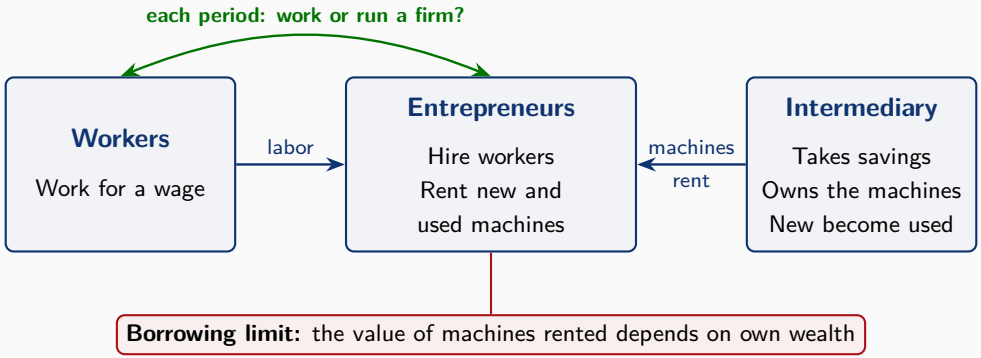
As young firms build wealth, they shift toward new machines.

▸ Regression ▸ Results

Model

Who Is in the Model?

People are heterogeneous in **wealth**, **entrepreneurial productivity**, and **worker ability**.



▸ Shocks

Preferences and Timing

- People live forever; each faces their own ups and downs, with no economy-wide shocks
- Each period, a person chooses how much to **consume** today, c, and how much to **save**, a'

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

β: **Patience**	How much a person values future consumption. $\Rightarrow$ Higher β: more patient, saves more.
σ: **Risk aversion**	How much a person dislikes risk. $\Rightarrow$ Higher σ: saves more for bad times.

Workers and entrepreneurs have the same preferences and both save.

Technology

An entrepreneur with productivity z hires labor L and rents new and used machines:

$$Y = \exp(z)\left(L^{\alpha}K^{1-\alpha}\right)^{\eta}, \qquad K = \left[\phi^{1/\gamma}K_n^{\frac{\gamma-1}{\gamma}} + (1-\phi)^{1/\gamma}K_u^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$$

$\eta < 1$: **Span of control**	Diminishing returns to managing a larger firm. $\Rightarrow$ Each firm has a best size; more talent, bigger firm.
γ: **Substitutability**	How easily a firm can use used machines in place of new ones. $\Rightarrow$ Higher γ: easier to switch to used when money is tight.
ϕ: **Weight on new**	How much new machines count in K (used: $1-\phi$). $\Rightarrow$ Higher ϕ: new adds more to output than used.

Firms choose the mix of machines, not only the amount.

▸ Talent z

Worker's Problem

- A worker with ability y supplies y efficiency units of labor.
- The wage per efficiency unit is W, so current labor income is:

$$I_w(y) = Wy.$$

- The worker chooses current consumption c and next period's assets a':

$$c + a' = Wy + (1 + r)a, \qquad a' \geq 0.$$

- Workers can save in deposits and earn the interest rate r, but they cannot borrow.
- People can switch between work and entrepreneurship every period

A worker's value includes both today's wage income and the option to become an entrepreneur later.

▸ Ability y

Entrepreneur's Problem

- The entrepreneur maximizes operating profit:

$$\pi(a,z) = \max_{K_n, K_u, L \geq 0} \{Y - WL - p_n R_n K_n - p_u R_u K_u\}.$$

- **Financing constraint:**

$$\underbrace{p_n K_n}_{\text{new capital value}} + \underbrace{p_u K_u}_{\text{used capital value}} \leq \underbrace{\theta a}_{\text{financing capacity}}$$

- Wealth a is collateral: the machines an entrepreneur rents can be worth at most θa at market prices

▸ Closed form ▸ Multiplier

Consumption, Saving, and Occupational Choice

Occupational choice

$$\Omega(a, z, y) = \max \left\{ V_e(a, z, y),\ V_w(a, z, y) \right\}$$

Same saving problem for both occupations $o \in \{e, w\}$:

$$V_o(a, z, y) = \max_{c,\ a' \geq 0} \left\{ u(c) + \beta\, E\big[\Omega(a', z', y') \mid z, y\big] \right\}$$

$$\text{s.t.} \quad c + a' = \underbrace{I_o}_{\text{only difference}} + (1 + r)a$$

Entrepreneur

$I_e = \pi(a, z)$

profits depend on wealth

Worker

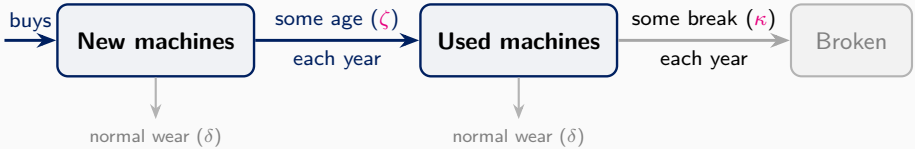
$I_w = Wy$

wage $\times$ worker ability

▸ Household problem

Where Used Capital Comes From

An **intermediary** owns all machines: it buys new ones and rents out both new and used



Proposition 1, in words

Rents just cover the intermediary's costs, so:

- a used machine has **few years left** before it breaks $\Rightarrow$ **worth less today**
- its rent must pay back its value in those few years $\Rightarrow$ **higher rent for its value**

Used machines are worth less, but cost more to rent for their value.

▸ Problem ▸ Proof ▸ Prices

Proposition 2: What Used Capital Does

Shadow cost μ: how much one more \$ of borrowing capacity is worth to the firm

zero with enough wealth; higher the more constrained the firm

Each machine really costs: **rent** + **shadow cost** $\times$ **its price**

Proposition 2, in words

The **higher the shadow cost**, the more the price matters relative to the rent:

- **Shadow cost** $= 0$: only rent matters $\Rightarrow$ mostly **new** machines
- **Higher shadow cost**: cheaper price wins $\Rightarrow$ more **used** machines
  $\Rightarrow$ more machines from the **same wealth**, up to a limit

$\Rightarrow$ **Bigger firms and more entry by talented but poor people**

▸ The mix ▸ More machines ▸ Statement ▸ Proof ▸ Price comparisons

The Goal: How Much Productivity Is Lost?

- **Productivity:** how much output the economy gets from the workers and machines it has
- **Why aggregate:** borrowing limits put inputs in the **wrong hands**, so the whole economy produces less from the same inputs
- **Formally**, aggregate TFP is output per unit of inputs (summed over all firms):

$$\text{TFP} = \frac{Y}{(L^{\alpha}K^{1-\alpha})^{\eta}}$$

- **TFP loss:** vs. the same economy **without a borrowing limit**, $\text{loss} = 100 \log \frac{\text{TFP}^{\text{no limit}}}{\text{TFP}^{\text{with limit}}}$

Question: how much smaller is the loss with used capital than without it?

▸ Equilibrium ▸ Market clearing ▸ Definitions

Quantitative Results

Calibration: Vietnamese

Parameter	Target moment	Data	What it says about Vietnam
θ: borrowing limit	Debt-to-output	0.600	how much firms can borrow
η_p: talent tail	Top-10% employment share	0.632	a few large firms hire most workers
ω: talent persistence	Firm exit rate	0.098	1 in 10 firms closes each year
σ_y: wage shocks	Worker wage Gini	0.372	how unequal wages are
β: patience	Real interest rate	0.031	a low return on savings
ζ: new becomes used	New-to-used ratio	2.570	many machines in use are second-hand
κ: used breaks	Used share vs. assets	-0.027	poorer firms use more used machines

Fit: average error across the seven moments is **0.3%**

Comparison economy: no used capital, recalibrated to the same common moments

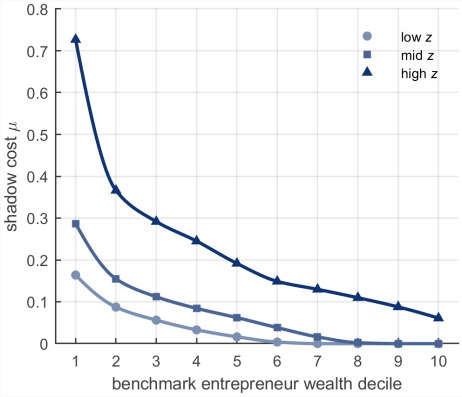
Seven parameters, seven moments: the model matches all of them.

▸ Targets and fit ▸ Used-capital parameters

Mechanism in the Calibrated Economy

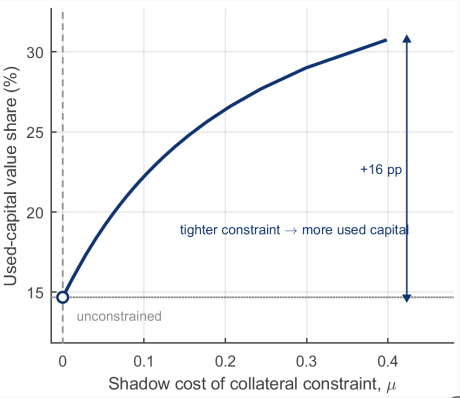
Who is constrained?

poor, productive firms have the highest μ



How do they respond?

higher $\mu \Rightarrow$ more used capital



▸ Numbers

Result 1: More Talented but Poor People Start Firms

	Used capital	No used capital	Difference
Talented, poor	42.0%	34.3%	$+7.7$ pp
Talented, rich	82.7%	83.3%	-0.6 pp

- Used machines let more **talented but poor** agents start a firm.
- Borrowing constraints keep fewer of them out: the entry gap falls by **23%**.
- Talented rich agents enter anyway: the constraint may limit their scale, but not their entry.

Who runs a firm depends less on wealth and more on talent.

▸ Details

Result 2: Less Misallocation

	Used capital	No used capital	Effect
Productivity (TFP) loss from frictions	5.72	9.39	39% lower
Gaps in returns to capital (sd log MRPK)	0.450	0.598	25% lower

Two channels, together:

- **more talented people run firms**
- **constrained firms get more capital** from the same wealth

Each economy is compared with its own frictionless version.

Used capital removes 39% of the TFP loss from financial frictions.

▸ Definitions

Conclusion

Takeaway

- Constrained firms adjust **which** capital they use, not only how much.
- Used machines are costlier to rent but use **less of the financing limit**.
- In Vietnamese SMEs, firms rely less on used equipment as their own assets grow.
- Access to used capital removes **39%** of the TFP loss from borrowing constraints and expands entry by talented, poor entrepreneurs.

Second-hand equipment markets partly substitute for missing credit.

▸ Robustness ▸ Designs ▸ CES parameters ▸ Scope

Thank You!

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Backup Slides

Evidence

- Fact 1: regression / results
- Fact 2: regression / results
- Fact 2 timing: regression / results
- Fact 2 robustness
- Zero debt / additional checks
- Fact 3: regression / results

Model

- Idiosyncratic shocks (z, y)
- Household problem
- Firm problem in closed form / the multiplier

Intermediary (Proposition 1)

- The intermediary's problem / proof
- Price vs. user cost; zero profit

Mechanism (Proposition 2)

- The mix: shadow cost and FOCs
- Collateral productivity Λ
- Proposition 2: statement / proof sketch
- Three price comparisons

Equilibrium and measurement

- Stationary equilibrium / market clearing
- Loss measures and definitions

Calibration and results

- Targets, sources and fit
- Identifying the used-capital parameters
- Proposition 2 at benchmark prices
- Entry shortfall by wealth group

Robustness and scope

- One calibration? / designs / ϕ and γ
- What the model abstracts from

Fact 1: Regression

$$100 \times \text{Constrained}_{it} = \beta_1 \, \text{Age}_{it}^{6\text{–}10} + \beta_2 \, \text{Age}_{it}^{11+} + \beta_3 \log \text{Assets}_{it} + \delta_{p(i),t} + \eta_{s(i)} + \varepsilon_{it}$$

Constrained

- They **applied** and reported a problem getting the loan, or their application was denied
- They **did not apply** because of inadequate collateral or a difficult process
- Not constrained: did not need a loan, did not want debt, or deterred by the interest rate or..

Fixed effects

- Province $\times$ year ($\delta_{p,t}$) and sector (η_s)

Comparison

- Between firms in the same province, year and sector; baseline: firms aged **0–5 years**

Fact 1: Results

	Credit constrained (all firms)		Put off applying (collateral or procedures)
	(1)	(2)	(3)
Age 6–10	$\mathbf{-3.41}^{***}$	$\mathbf{-3.33}^{***}$	$\mathbf{-4.39}^{***}$
	(1.15)	(1.14)	(1.38)
Age 11+	$\mathbf{-6.79}^{***}$	$\mathbf{-6.26}^{***}$	$\mathbf{-7.75}^{***}$
	(1.09)	(1.09)	(1.31)
Log assets		$\mathbf{-1.25}^{***}$	$\mathbf{-1.41}^{***}$
		(0.28)	(0.31)
Province × year, sector FE	Yes	Yes	Yes
Observations	14,587	14,562	9,674
Firms	4,889	4,887	4,017

S.e. clustered by firm. $^{***}p < 0.01$. Age bands relative to age 0–5.

Back

Fact 2: Regression

(1) $\text{Used}_{it} = \chi\, \text{Constrained}_{it} + \Gamma' Z_{it} + \nu_i + \lambda_t + \varepsilon_{it}$

(2) $\text{Used}_{it} = \chi_1\, \text{Constrained}_{it} \times \text{Debt}_{it} + \chi_2\, \text{Constrained}_{it} \times \text{NoDebt}_{it} + \psi\, \text{Debt}_{it} + \Gamma' Z_{it} + \nu_i + \lambda_t + \varepsilon_{it}$

(3) $\text{Used}_{it} = \chi_1\, \text{CollateralProblem}_{it} + \chi_2\, \text{OtherProblem}_{it} + \Gamma' Z_{it} + \nu_i + \lambda_t + \varepsilon_{it}$

Constrained, in (1) and (2)

- **Applied** and reported a problem getting the loan, or the application was denied
- **Did not apply** because of inadequate collateral or a difficult process
- **Baseline:** the same firm in rounds when it is not constrained

(2): split by debt

- Constrained × Debt: has a loan but **cannot borrow more**
- Constrained × NoDebt: constrained with no loan
- **Baseline:** unconstrained rounds with the same debt status

(3): only firms that applied for a loan

- CollateralProblem = 1: problem due to **lack of collateral**
- OtherProblem = 1: problem for another reason
- **Baseline:** applicants who reported **no problem**

Back

Fact 2: Results

	Used share (pp)		
	(1)	(2)	(3)
Constrained	**1.72**** (0.86)		
Constrained × debt		**2.75***** (1.01)	
Constrained × no debt		−0.61 (1.61)	
Has debt		−0.16 (0.77)	
Problem: lack of collateral			**5.95**** (2.63)
Problem: other reasons			1.42 (1.76)
Log employment	−0.82 (0.75)	Yes	Yes
Log assets	−0.71* (0.40)	Yes	Yes
Age 6–10	−1.11 (1.37)	Yes	Yes
Age 11+	−1.88 (1.86)	Yes	Yes
Firm FE, year FE	Yes	Yes	Yes
Observations	13,197	13,197	3,714
Firms	3,487	3,487	1,265

(1) all firms; (2) by debt; (3) loan applicants vs. applicants with no problems. Mean used share 24.0. S.e. clustered by firm. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

◂ Back

Fact 2 Timing: Regression

$$\Delta \text{Used}_{it} = \beta_1 \, \text{Became}_{it} + \beta_2 \, \text{Stopped}_{it} + \beta_3 \, \text{Becomes}_{i,t+1} + \beta_4 \, \text{Stops}_{i,t+1} + \Gamma' \Delta Z_{it} + \lambda_t + \Delta \varepsilon_{it}$$

- **Became / Stopped**: firm becomes or stops being constrained *this* round
- **Placebo**: status change in the *next* round. If firms moved to used equipment before the constraint, $\beta_3 > 0$
- Differencing removes the firm effect

The shift to used equipment should come with the constraint, not before it.

Back

Fact 2 Timing: Results

	Δ Used (1)	Δ Any used (2)	Δ Used (3)	Δ Any used (4)
Became constrained	1.87	**3.61****	**5.11****	**7.77****
	(1.35)	(1.80)	(2.48)	(3.33)
Stopped being constrained	−1.37	−2.37	−1.49	−3.15
	(1.18)	(1.59)	(1.58)	(2.13)
Placebo: becomes next round			−0.77	−2.47
			(1.69)	(2.31)
Placebo: stops next round			−2.96	−4.46
			(2.27)	(3.01)
Year FE	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes
Pairs of rounds	9,524	9,524	6,027	6,027
Firms	3,459	3,459	2,437	2,437

S.e. clustered by firm. $^{**}p < 0.05$. The placebo is not positive: no sign that firms shift to used equipment before reporting a constraint.

Back

Fact 2 Robustness: Results

Same regression as Fact 2, column (1). Each row changes one thing; each cell is χ.

	Used share	Any used	Obs. / firms
(a) Baseline	1.72^{**} (0.86)	2.83^{**} (1.13)	13,197 / 3,487
(b) Excluding 2015	2.05^{**} (0.97)	3.10^{**} (1.26)	10,806 / 3,116
(c) Rounds 2007–2015	2.34^{**} (0.93)	3.44^{***} (1.25)	10,758 / 3,080
(d) Firms observed in 3+ rounds	2.05^{**} (0.90)	2.97^{**} (1.19)	11,225 / 2,501
(e) Firms that did not change owner	1.66^{*} (0.86)	2.83^{**} (1.14)	12,835 / 3,438
(f) Reported problems only	1.74^{**} (0.86)	2.88^{**} (1.13)	13,195 / 3,487
(g) Size classes instead of log employment	1.71^{**} (0.86)	2.82^{**} (1.13)	13,197 / 3,487
(h) No asset control	1.72^{**} (0.86)	2.82^{**} (1.13)	13,199 / 3,487
(i) No age controls	1.73^{**} (0.86)	2.83^{**} (1.13)	13,206 / 3,488
(j) Log age instead of age bands	1.74^{**} (0.86)	2.84^{**} (1.13)	13,197 / 3,487
(k) Sector $\times$ year + province $\times$ year FE	1.46^{*} (0.88)	2.58^{**} (1.15)	13,160 / 3,485
(l) Clustered by province $\times$ sector	1.77^{*} (1.04)	2.88^{**} (1.28)	13,160 / 3,485

S.e. in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Back

Zero Debt and Used Capital

$$\text{Used}_{it} = \chi_1 \,\text{ZeroDebt}_{it} + \chi_2 \,(\text{ZeroDebt}_{it} \times X_{it}) + \chi_3 X_{it} + \Gamma' Z_{it} + \nu_i + \lambda_t + \varepsilon_{it}$$

	Used share (pp)	
	X = log assets	X = young
Zero debt $\times$ X	**−1.07**** (0.46)	**3.89*** (2.34)
Firm FE, year FE, size controls	Yes	Yes
Observations	13,492	13,403

- Without debt, the used share falls faster with assets and is higher for young firms
- But most zero-debt firms **did not need a loan**: zero debt is not a constraint

So the main test uses the direct credit-constraint measure.

S.e. clustered by firm. $^{**}p < 0.05$, $^{*}p < 0.10$.

Back

Empirics: Additional Checks

Direct reports of binding constraints (formal loan applicants)

- Applicants reporting loan problems hold **11.7 pp** more used capital in 2013 (s.e. 3.10, $N = 644$); pooled 2011–2015, **6.8 pp** (s.e. 1.82, $N = 2{,}010$); similar with controls

Column (2): young $\times$ zero-debt interaction

- Raw reported age: 5.345 (2.378); firms observed in ≥ 3 rounds: 4.801 (2.439)
- Within-firm variation: ZeroDebt switches for 2,143 of 3,549 firms, Young for 1,021

Reading the evidence

- ZeroDebt marks self-financing, not rationing: most non-applicants did not need or want credit; 2.6% cite inadequate collateral
- No unconditional within-firm age gradient: Young $= 0.755$ (1.658) for firms with debt
- Attrition and the stock (not flow) nature of the vintage item work *against* finding an effect, and align the regressions with the model's value share

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Fact 3: Regression

$$y_{it} = \xi \left(\text{Young}_i \times \tau_{it} \right) + \nu_i + \lambda_t + \varepsilon_{it}$$

- **Young**$_i$: firm first observed at age 0–5 (1,509 firms)
- τ_{it}: rounds since the firm was first observed (one round = 2 years)
- ξ: how much faster a young firm's outcome changes every 2 years than an older firm's
- **Outcomes**: used share, log assets, log employment
- Firm and year fixed effects: **within firm**, young vs. older firms over the same years

If constraints loosen as firms grow, young firms should leave used equipment behind.

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Fact 3: Results

	Used share (1)	Used rel. to new (2)	Any used (3)	Log assets (4)	Log empl. (5)
Young × rounds since first observed	-1.28^{**}	-1.20^{**}	-1.40^{**}	7.97^{***}	1.82^{**}
	(0.53)	(0.54)	(0.68)	(1.66)	(0.85)
Firm FE / Year FE	Yes	Yes	Yes	Yes	Yes
Observations	13,279	13,062	13,279	13,277	13,256
Firms	3,494	3,468	3,494	3,494	3,492
Mean of dep. var.	24.1	25.3	35.4	—	—

Cols. (4)–(5) are $100\times$log. After 10 years (event study, Figure 3): used share -6.4 pp, log assets $+37$. S.e. clustered by firm. $^{***}p < 0.01$, $^{**}p < 0.05$.

Used-share coefficient under other specifications: young ≤ 3: -1.66^{**}; young ≤ 7: -1.42^{***}; province×year FE: -0.96^{*}; excl. 2015: -1.48^{**}; first seen 2005: -1.30^{**}; with size and credit controls: -1.21^{**}.

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Idiosyncratic Shocks

Entrepreneurial productivity (persistent, with occasional redraws):

$$z' = \begin{cases} z, & \text{with probability } \omega, \\ \tilde{z}', \quad \tilde{z}' \sim F_z, & \text{with probability } 1-\omega, \end{cases} \qquad e^z \text{ has a Pareto right tail with parameter } \eta_p$$

Worker ability (AR(1) in logs, independent of z):

$$\log y' = \rho_y \log y + \sigma_y \, \varepsilon_y', \qquad \varepsilon_y' \sim N(0,1), \qquad \mathrm{sd}(\log y) = \frac{\sigma_y}{\sqrt{1-\rho_y^2}}$$

What disciplines each parameter

- $\omega \leftrightarrow$ firm exit rate 0.098 (switches from entrepreneurship to wage work)
- $\eta_p \leftrightarrow$ employment share of the top decile of firms, 0.632
- $\sigma_y \leftrightarrow$ worker wage Gini, 0.372; $\rho_y = 0.95$ assigned (Cagetti & De Nardi 2006)

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Household Problem and Occupational Choice

Preferences: $U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad u(c) = \dfrac{c^{1-\sigma} - 1}{1-\sigma}$

Values as an entrepreneur and as a worker:

$$V_e(a,z,y) = \max_{c\geq 0,\ a'\geq 0} \Big\{ u(c) + \beta\, \mathbb{E}\big[\Omega(a',z',y') \,\big|\, z,y\big] \Big\} \quad \text{s.t.} \quad c + a' = \pi(a,z) + (1+r)\,a$$

$$V_w(a,z,y) = \max_{c\geq 0,\ a'\geq 0} \Big\{ u(c) + \beta\, \mathbb{E}\big[\Omega(a',z',y') \,\big|\, z,y\big] \Big\} \quad \text{s.t.} \quad c + a' = W y + (1+r)\,a$$

Occupational choice: $\Omega(a,z,y) = \max\big\{V_e(a,z,y),\, V_w(a,z,y)\big\}$; entrepreneur $\iff V_e \geq V_w$

- $a' \geq 0$: no borrowing against labor income; the only credit is intermediated capital, limited by $p_n K_n + p_u K_u \leq \theta a$
- Occupation is a function of the current state, not a coordinate of the distribution $G(a,z,y)$
- No exogenous exit: the survey's exit rate maps to switches from running a firm to wage work

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Firm Problem in Closed Form (1/2): Given μ

Lagrangian of the firm's problem:

$$\mathcal{L} = e^z \left(L^\alpha K^{1-\alpha}\right)^\eta - WL - p_n R_n K_n - p_u R_u K_u + \mu \left(\theta a - p_n K_n - p_u K_u\right)$$

For a given μ, an unconstrained problem with **effective unit prices**

$$c_j(\mu) = p_j \left(R_j + \mu\right), \qquad j \in \{n, u\}$$

CES cost minimization: price of one unit of effective capital and conditional demands

$$P_K(\mu) = \left[\phi\, c_n(\mu)^{1-\gamma} + (1-\phi)\, c_u(\mu)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \qquad K_n = \phi\left(\frac{c_n}{P_K}\right)^{-\gamma} K, \quad K_u = (1-\phi)\left(\frac{c_u}{P_K}\right)^{-\gamma} K$$

Scale, labor, and output (decreasing returns, $\eta < 1$):

$$K^d(\mu) = \left[e^z \left(\frac{\alpha\eta}{W}\right)^{\alpha\eta}\right]^{\frac{1}{1-\eta}} \left[\frac{(1-\alpha)\eta}{P_K(\mu)}\right]^{\frac{1-\alpha\eta}{1-\eta}}, \qquad L = \frac{\alpha\eta\, Y}{W}, \qquad \frac{(1-\alpha)\eta\, Y}{K} = P_K(\mu)$$

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Firm Problem in Closed Form (2/2): The Multiplier

Collateral used at μ is $K^d(\mu)/\Lambda(\rho(\mu))$. With $\Lambda^{uc} \equiv \Lambda(\rho(0))$, complementary slackness gives

$$\mu = 0 \;\text{ if }\; K^d(0) \le \Lambda^{uc}\,\theta a; \qquad \text{otherwise } \mu > 0 \text{ solves } \quad K^d(\mu) = \Lambda\big(\rho(\mu)\big)\,\theta a$$

$K^d(\mu)$ falls with μ (P_K rises) and $\Lambda(\rho(\mu))$ rises (Prop. 2) $\Rightarrow$ a unique root, found by bisection.

Policies when the constraint binds:

$$K_n = \frac{\theta a}{p_n + p_u\,\rho(\mu)}, \qquad K_u = \rho(\mu)\,K_n, \qquad K = \Lambda\big(\rho(\mu)\big)\,\theta a, \qquad L = \left[\frac{\alpha\eta\, e^z K^{(1-\alpha)\eta}}{W}\right]^{\frac{1}{1-\alpha\eta}}$$

$$\pi(a,z) = e^z\left(L^\alpha K^{1-\alpha}\right)^\eta - W L - p_n R_n K_n - p_u R_u K_u$$

- Marginal revenue product of effective capital $= P_K(\mu)$: **MRPK differs across firms only through** μ
- Two margins: scale (how far θa reaches) and mix ($\rho(\mu)$, how far each dollar stretches)

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The Intermediary's Problem

The intermediary discounts at r, takes prices as given, and chooses investment:

$$V(K_n, K_u) = \max_{K_n', I} \left\{ R_n K_n + p_u R_u K_u - I + \frac{V(K_n', K_u')}{1+r} \right\}$$

$$\text{s.t.} \qquad K_n' = (1-\delta-\zeta)\, K_n + I, \qquad K_u' = (1-\delta-\kappa)\, K_u + \zeta\, K_n$$

Proposition 1 (capital prices)

A stationary solution with $K_n^s, K_u^s \in (0, \infty)$ requires

$$R_n = r + \delta + \zeta \left(1 - \frac{p_u R_u}{r+\delta+\kappa}\right)$$

- Only the product $p_u R_u$ enters this condition and any rental bill; the collateral constraint, written on market value, is what makes p_u and R_u matter separately
- Deposits are the only liability, capital the only asset: $D = p_n K_n^s + p_u K_u^s$

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Proof of Proposition 1

Envelope conditions and the first-order condition for K_n' (primes denote next period):

$$V_n = R_n + (1-\delta-\zeta) + \frac{\zeta}{1+r}\, V_u' \tag{A2}$$

$$V_u = p_u R_u + \frac{1-\delta-\kappa}{1+r}\, V_u' \tag{A3}$$

$$0 = -1 + \frac{V_n'}{1+r} \quad \Longrightarrow \quad V_n' = 1+r \tag{A4}$$

In (A2), the continuation term $(1-\delta-\zeta)\, V_n'/(1+r)$ has already been replaced by $1-\delta-\zeta$ using (A4).

Iterate (A3) forward and impose the steady state:

$$V_u = p_u R_u \sum_{s \geq 0} \left(\frac{1-\delta-\kappa}{1+r} \right)^s = p_u R_u \, \frac{1+r}{r+\delta+\kappa} \tag{A5}$$

Substitute (A4) and (A5) into (A2):

$$1+r = R_n + 1 - \delta - \zeta + \zeta \, \frac{p_u R_u}{r+\delta+\kappa} \quad \Longrightarrow \quad R_n = r + \delta + \zeta \left(1 - \frac{p_u R_u}{r+\delta+\kappa} \right) \qquad \blacksquare$$

Splitting Price and User Cost; Zero Profit

No arbitrage on a used unit. Buy it at p_u, earn the rent $p_u R_u$, keep $(1-\delta-\kappa)\,p_u$:

$$(1+r)\,p_u = p_u R_u + (1-\delta-\kappa)\,p_u \quad \Longrightarrow \quad R_u = r+\delta+\kappa$$

The rent covers interest, depreciation, and breakage. Then $V_u = (1+r)\,p_u$, and Proposition 1 becomes

$$R_n = r+\delta+\zeta\,(p_n - p_u) \quad \Longleftrightarrow \quad p_u = p_n - \frac{R_n - r - \delta}{\zeta}$$

Zero profit. The technology is linear, so V is homogeneous of degree one. By Euler's theorem,

$$V = V_n K_n^s + V_u K_u^s = (1+r)\left(p_n K_n^s + p_u K_u^s\right) = (1+r)\,D$$

The intermediary is a veil: the economy behaves as if households held the capital stock at prices (p_n, p_u).

Price adjustment. A higher R_n lowers p_u, which tilts demand toward used capital through both the rent and the collateral a unit absorbs.

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The Mix of Machines: The Math

Goal: how does the financing constraint change the mix of new and used machines?

- Entrepreneur's problem with multiplier $\mu(a,z) \geq 0$ on the financing constraint:

$$\mathcal{L} = Y - WL - p_n R_n K_n - p_u R_u K_u + \mu(a,z)\left(\theta a - p_n K_n - p_u K_u\right)$$

- $\mu(a,z) =$ **shadow cost**: extra profit from \$1 more financing capacity (how tight collateral is).
- First-order conditions: each machine costs its rent plus the capacity it uses

$$MPK_n = p_n(\underbrace{R_n}_{\text{rent}} + \underbrace{\mu}_{\text{capacity}}), \qquad MPK_u = p_u(R_u + \mu).$$

- Dividing them gives the mix $\rho \equiv K_u/K_n$:

$$\rho(\mu) = \frac{1-\phi}{\phi}\left[\frac{p_n(R_n+\mu)}{p_u(R_u+\mu)}\right]^{\gamma}$$

  - $\mu = 0$ (**rich firm**): compares *rents* $p_n R_n$ vs. $p_u R_u \Rightarrow$ same mix ρ^{uc} for all.
  - μ large (**poor firm**): compares *collateral* p_n vs. $p_u \Rightarrow$ more used capital.

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Collateral Productivity: The Math

Goal: what does a constrained firm gain from the mix of new and used machines?

- **Collateral productivity** Λ = capital per dollar of collateral:

$$\Lambda = \frac{K}{p_n K_n + p_u K_u}$$

- For a constrained firm, the financing constraint binds:

$$p_n K_n + p_u K_u = \theta a$$

- Multiply both sides by Λ:

$$K = \Lambda \left(p_n K_n + p_u K_u \right) \quad \Rightarrow \quad K = \underbrace{\Lambda}_{\text{mix}} \underbrace{\theta a}_{\text{wealth}}$$

- Wealth fixes θa: the firm cannot change it today.
- The choice of new vs. used decides Λ: the firm *can* change it.

Same wealth, better mix $\Rightarrow$ more capital.

Proposition 2: Formal Statement

Goal: show that constrained firms tilt toward used capital, and what they gain from it.

Proposition 2 (Used-capital tilt and collateral relief)

If $R_u > R_n$ (equivalently $\kappa > \zeta\,(p_n - p_u)$), then the more constrained the firm (higher $\mu(a, z)$):

- it uses **more used capital** relative to new;
- it gets **more capital per dollar of collateral** (Λ rises), but the gain is bounded.

Intensive margin
constrained firms run at a larger scale

Extensive margin
more poor, productive people enter

Proposition 2: Proof Sketch

(i) Tilt. For given effective capital K the firm solves

$$\min_{K_n, K_u} \; p_n(R_n+\mu)K_n + p_u(R_u+\mu)K_u \quad \text{s.t.} \quad \left[\phi^{\frac{1}{\gamma}} K_n^{\frac{\gamma-1}{\gamma}} + (1-\phi)^{\frac{1}{\gamma}} K_u^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} = K$$

The FOCs give $\rho(\mu) = \bar{\rho}\,[(R_n+\mu)/(R_u+\mu)]^{\gamma}$, and

$$\frac{\partial}{\partial \mu}\frac{R_n+\mu}{R_u+\mu} = \frac{R_u - R_n}{(R_u+\mu)^2} > 0 \iff R_u > R_n, \qquad \frac{R_n+\mu}{R_u+\mu} \xrightarrow[\mu\to\infty]{} 1 \;\Rightarrow\; \rho(\mu) \to \bar{\rho}$$

(ii) Relief. Maximizing $\Lambda(\rho) \iff$ minimizing collateral per unit of the bundle, $(p_n + p_u\rho)/\Psi(\rho)$:

- Strictly quasi-convex: $\rho \mapsto p_n + p_u\rho - t\,\Psi(\rho)$ is strictly convex for every $t > 0$ (Ψ strictly concave) $\Rightarrow$ unique minimizer $\bar{\rho}$ (user costs drop out as $\mu \to \infty$)
- $\rho(\mu)$ rises from $\rho^{uc} < \bar{\rho}$ toward $\bar{\rho}$, and Λ is increasing on $(0, \bar{\rho}) \Rightarrow \Lambda(\rho(\mu)) \uparrow \bar{\Lambda}$
- $\bar{\Lambda} > 1/p_n$ when $p_u < p_n$, $\gamma > 1$: then $p_u^{1-\gamma} > p_n^{1-\gamma}$, so the CES mean exceeds $p_n^{1-\gamma}$

Condition. $R_u > R_n \iff \kappa > \zeta(p_n - p_u)$; sufficient: $\zeta \le \kappa \iff K_u^s/K_n^s \le \kappa/(\delta+\kappa)$

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Three Price Comparisons: Which One the Mechanism Uses

Comparison	Condition	What it delivers
Price per unit	$p_u < p_n \iff R_n > r + \delta$	$\bar{\Lambda} > 1/p_n$: more capital per collateral dollar
User cost per \$ of value	$R_u > R_n \iff \kappa > \zeta\,(p_n - p_u)$	the tilt, $\rho'(\mu) > 0$
Rent per unit	$p_u R_u \gtrless p_n R_n$	nothing: no result depends on it

Read the effective price $p_j(R_j + \mu)$ per rental dollar. A dollar of rent on type j ties up

$$\frac{p_j}{p_j R_j} = \frac{1}{R_j}$$

of financing capacity. The price cancels, so on this margin **the user cost alone decides**: renting used capital frees collateral relative to new exactly when $R_u > R_n$.

At the calibration: $p_u = 0.645 < 1$, $R_u = 0.234 > R_n = 0.119$, and $p_u R_u \approx 0.151 > p_n R_n = 0.119$. Both $p_u < p_n$ and $R_u > R_n$ are equilibrium properties, not restrictions on primitives.

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Stationary Competitive Equilibrium

Prices (W, r, R_n) with implied $p_u = p_n - (R_n - r - \delta)/\zeta$ and $R_u = r + \delta + \kappa$; value functions $\{V_e, V_w, \Omega\}$; policies; firm input demands; stocks (K_n^s, K_u^s); and a distribution $G(a, z, y)$ such that:

1. given (W, R_n, R_u, p_u), firm input demands solve the profit problem subject to the collateral constraint;
2. given prices and Ω, consumption, saving, and occupation solve the household problem;
3. the intermediary's investment is optimal given (r, R_n): Proposition 1 holds, profit is zero, and $D = p_n K_n^s + p_u K_u^s$;
4. G is invariant under the optimal policies and the processes for z and y;
5. the labor, new-capital, and used-capital markets clear.

Benchmark	W	r	R_n	p_u	R_u	Entrepreneurs	Constrained
Equilibrium	0.540	0.031	0.119	0.645	0.234	0.221	0.908

Aggregation and Market Clearing

Entrepreneurs and workers: $\mathcal{E} = \{(a,z,y) : V_e \geq V_w\}$, $\mathcal{W} = \{(a,z,y) : V_e < V_w\}$

$$K_n^d = \int_{\mathcal{E}} K_n(a,z)\, dG, \qquad K_u^d = \int_{\mathcal{E}} K_u(a,z)\, dG, \qquad L^d = \int_{\mathcal{E}} L(a,z)\, dG, \qquad L^s = \int_{\mathcal{W}} y\, dG$$

Supply: stationary used stock and the balance sheet, with $D = \int a\, dG$:

$$\zeta K_n^s = (\delta + \kappa)\, K_u^s, \quad D = p_n K_n^s + p_u K_u^s \quad \Longrightarrow \quad K_n^s = \frac{D}{p_n + p_u\, \zeta/(\delta+\kappa)}, \quad K_u^s = \frac{\zeta}{\delta+\kappa}\, K_n^s$$

Market clearing: $K_n^d = K_n^s$, $K_u^d = K_u^s$, $L^d = L^s$. Three prices, three markets:

- W clears labor; r clears new capital through deposits; R_n adjusts until $K_n^d/K_u^d = (\delta+\kappa)/\zeta$, after which used-capital clearing is automatic

External finance (the moment that disciplines θ):

$$B^{ext} = \int_{\mathcal{E}} \max\big\{0,\ p_n K_n(a,z) + p_u K_u(a,z) - a\big\}\, dG, \qquad B^{ext}/Y = 0.600$$

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Loss Measures and Definitions

Frictionless counterpart. In each economy, raise θ until the constraint is slack for every operating firm, hold all other parameters fixed, and recompute prices, occupations, and the stationary distribution. MRPK dispersion is then numerically zero.

$$\mathcal{M} = 100 \log \frac{\text{TFP}^{\text{frictionless}}}{\text{TFP}^{\text{friction}}}, \qquad \text{share removed} = \frac{\mathcal{M}^{\text{no used}} - \mathcal{M}^{\text{bm}}}{\mathcal{M}^{\text{no used}}} = \frac{9.39 - 5.72}{9.39} = 39.1\%$$

$$\mathcal{S} = 100 \log \frac{e^{\text{frictionless}}}{e^{\text{friction}}}, \qquad \text{share repaired} = 1 - \frac{\mathcal{S}^{\text{bm}}}{\mathcal{S}^{\text{no used}}} = 1 - \frac{53.2}{68.7} = 22.6\%$$

- e: entrepreneurship rate of poor talented agents (top decile of z, wealth below the benchmark median; cutoffs built once in the benchmark and applied in levels to both economies)
- K/K^*: a firm's new capital relative to what it would install if the constraint did not bind, at the economy's prices; averaged across firms, very high ratios capped
- MRPK* (Figure 2): the economy-wide average MRPK in the frictionless economy

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Calibration Targets and Fit

Moment	Param.	Data	Model	Source
Debt-to-output, B^{ext}/Y	θ	0.600	0.600	Debt over value added, 2013 & 2015 SME rounds
Top-decile employment share	η_p	0.632	0.633	2013 SME round, paid employment
Annual firm exit rate	ω	0.098	0.097	SME survey; cf. 9–10% in Berkel et al. (2020)
Worker wage Gini	σ_y	0.372	0.372	VHLSS 2010 (Doan et al. 2023)
Real interest rate	β	0.031	0.031	Deposit rate less GDP-deflator inflation, 2013 (WDI)
New-to-used capital ratio	ζ	2.570	2.577	SME survey
Used-share gradient in log assets	κ	-0.027	-0.027	SME survey, 2013 cross-section

- Model column: benchmark. Average error across the seven moments 0.31% (largest 0.61%). The no-used-capital economy matches the five common moments
- $\beta = 0.858$ is what clears the deposit market at $r = 3.1\%$ once entrepreneurs earn a return premium; close to the 0.865 in Cagetti & De Nardi (2006)
- Exit: the survey records closure or suspension of the household's enterprise, which is what a switch to wage work is in the model

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Identifying the Used-Capital Parameters

Relative quantity (stationary stocks):

$$\frac{K_n^s}{K_u^s} = \frac{\delta + \kappa}{\zeta} = 2.570$$

Relative collateral price
(firms with a positive used share, 2013):

$$s_{u,i} = b_0 + b_1 \log A_i + \varepsilon_i$$

$$b_1 = -0.027 \ (0.0061), \qquad N = 886$$

Jointly: $\kappa = 0.143$, $\zeta = 0.079$
$\Rightarrow R_u = 0.234 > R_n = 0.119$, and $\zeta \leq \kappa$ holds

Why the gradient pins κ

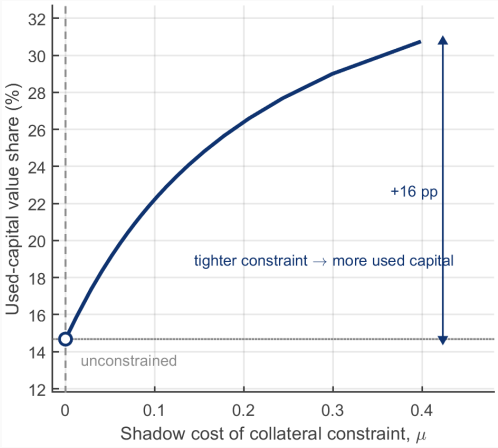
- A bigger collateral advantage of used capital $\Rightarrow$ small, constrained firms lean on it more $\Rightarrow$ a steeper (more negative) b_1
- A between-firm size gradient in one cross-section, natural for a stationary model; distinct from the within-firm interactions in Table 2

Assigned CES parameters

- $\phi = 0.70$ is weakly identified: ζ delivers the aggregate ratio at any ϕ. At $\phi = 0.75$ (Lanteri 2018) the results are unchanged
- $\gamma = 2$: Edgerton (2011) estimates 1.7–2.0 for farm and 1.9–2.4 for construction machinery

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Proposition 2 at Benchmark Prices



At $p_u = 0.645$, $R_n = 0.119$, $R_u = 0.234$, $\phi = 0.7$, $\gamma = 2$:

μ	K_u/K_n	Used value share	$\Lambda(\rho(\mu))$
0	0.267	14.7%	1.069
0.1	0.443	22.2%	1.122
0.2	0.557	26.4%	1.141
0.4	0.691	30.8%	1.155
∞	1.030	39.9%	1.165

- Steep at small μ: nearly half the rise to $\mu = 0.4$ is done by $\mu = 0.1$
- Bounded relief: Λ at most 9% above the unconstrained mix
- Same $\mu \Rightarrow$ same mix: a financing response, not technology

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Details: Entry Shortfall by Wealth Group

Entry shortfall: $\mathcal{S} = 100 \log\left(\frac{e^{\text{frictionless}}}{e^{\text{friction}}}\right)$ (how much the constraint lowers entry)

Talented = top-decile z	Used capital	No used capital
Poor (wealth below the median)		
Entry rate, with the constraint	0.420	0.343
Entry rate, frictionless	0.714	0.682
Shortfall $\mathcal{S}$ (log points)	53.2	68.7
Rich (wealth above the median)		
Entry rate, with the constraint	0.827	0.833
Entry rate, frictionless	0.801	0.744
Shortfall $\mathcal{S}$ (log points)	-3.2	-11.3

- **Poor:** used capital shrinks the entry gap by $1 - \frac{53.2}{68.7} = 23\%$.
- **Rich:** negative shortfall, a general-equilibrium wage effect. Removing the constraint raises labor demand and the wage, so some marginal high-z agents switch to wage work.

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Robustness: The Result Does Not Depend on One Calibration

Comparison economy without used capital	TFP loss	Gain from used capital
Productivity distribution held fixed	7.11	19.5%
Fully recalibrated (headline)	9.39	39.1%

- The quantitative size depends on the comparison: used capital removes **20–39%** of the TFP loss.
- It repairs **23–32%** of the lost entry by talented, poor agents.
- Across all specifications, the conclusion is unchanged:

Used capital raises scale, lowers misallocation, and expands entry by talented poor agents.

Comparison Designs: Full Results

Economy	η_p	$\mathcal{M}$ (log pts)	Misalloc. reduced	Entry repaired	K/K^*	sd(log MRPK)
Benchmark (used capital)	5.949	5.72	—	—	0.343	0.450
No used capital, tail pinned	5.949	7.11	19.5%	32.0%	0.268	0.530
Tail diagnostic	5.949	7.42	22.9%	27.8%	0.295	0.530
No used capital, recalibrated	5.397	9.39	39.1%	22.6%	0.279	0.598

- **Tail pinned:** $(\eta_p, \omega, \sigma_y)$ at benchmark values; only (θ, β) recalibrated
- **Tail diagnostic:** recalibrated economy with η_p back at 5.949; prices re-cleared, no moment refit
- Restoring η_p alone moves $\mathcal{M}$ from 9.39 to 7.42; $(\theta, \omega, \sigma_y, \beta)$ and prices account for the other 0.31
- The margins move in opposite directions as the tail is refit: a fatter tail adds very productive entrepreneurs, costly to constrain (misallocation share rises) but inframarginal for entry (entry share falls)

Misalloc. reduced $= 1 - \mathcal{M}^{\text{bm}}/\mathcal{M}$; entry repaired $= 1 - \mathcal{S}^{\text{bm}}/\mathcal{S}$; each economy against its own frictionless twin.

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Robustness to the Assigned CES Parameters

	Benchmark ($\phi = 0.70$, $\gamma = 2$)	$\phi = 0.75$	$\gamma = 1.7$	$\gamma = 2.5$
Gain from removing the collateral constraint				
TFP loss $\mathcal{M}$ (log points)	5.72	5.74	5.75	5.70
Misallocation reduced (%)	39.1	38.9	38.7	39.3
Allocation in the constrained equilibrium				
Mean scale, K/K^*	0.343	0.350	0.347	0.337
Share of firms constrained	0.908	0.906	0.907	0.908
Entrepreneur share	0.221	0.220	0.221	0.220
Cleared prices				
Wage W	0.540	0.542	0.540	0.540
Interest rate r	0.031	0.031	0.031	0.031
New-capital rental R_n	0.119	0.124	0.119	0.120
Used-capital price p_u	0.645	0.582	0.648	0.640

Each column re-clears (W, r, R_n) holding $(\theta, \eta_p, \omega, \kappa, \zeta, \sigma_y, \beta)$ at benchmark values: sensitivity sweeps, not recalibrations. $\mathcal{M}$ stays between 5.70 and 5.75 log points.

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What the Model Abstracts From

Deliberately outside the model

- Supply of used capital is reduced form: new capital ages at rate ζ, used capital breaks at rate κ, and the intermediary prices both at zero profit
- No resale decision by firms, and no illiquidity or adverse selection in the used market (cf. Lanteri 2018; Lanteri & Rampini 2023; Bierdel, Drenik, Herreño & Ottonello 2023)
- No timing-of-payments channel (Eisfeldt & Rampini 2007); firms rent capital services
- Stationary economy: no aggregate uncertainty, no transition dynamics

What the mechanism needs

- Collateral counted at **market value**: $p_n K_n + p_u K_u \leq \theta a$
- Used capital with a higher user cost per dollar of value: $R_u > R_n$

$\Rightarrow$ Under these two, Proposition 2 delivers the tilt and the collateral relief, whatever produces p_u and R_u

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